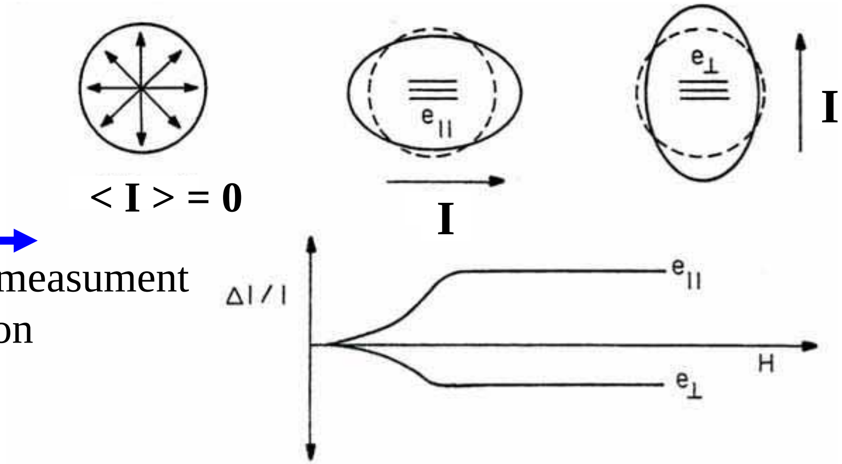
July 5, 2006

2.2 Magnetoelastic anisotropy

2.2.1 Anisotropic Magnetostriction

Demagnetized



Strain measument direction

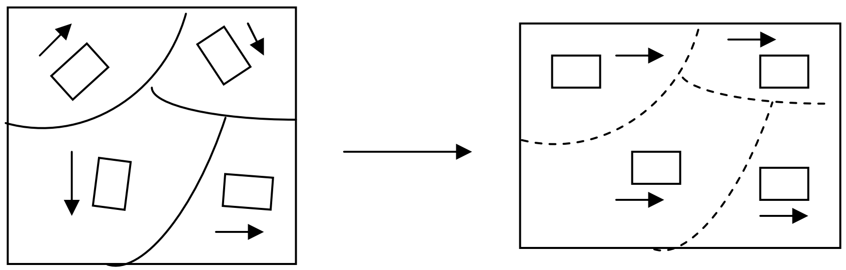
When an external field is applied, the sample strains anisotropically.

$\Delta l / l = \lambda,$ **Magnetostriction**

$\lambda \sim 10^{-7} - 10^{-4}$ for 3d metals and alloys

$\lambda > 10^{-3}$ for some 4f metals and alloys

Magnetostriction is understood by the domain structure change.



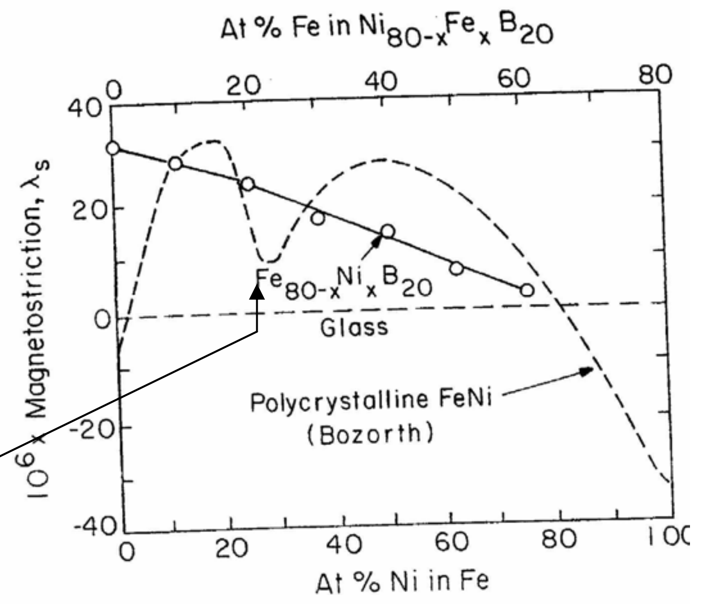
$H = 0$ $H = H_s$

Magnetostrictions for various magnetic materials

	$T = 4.2\,K$		Room Temperature			
	$\lambda_{100}(\lambda^{\gamma,2})$	$\lambda_{111}(\lambda^{\varepsilon,2})$	$\lambda_{100}(\lambda^{\gamma,2})$	$\lambda_{111}(\lambda^{\varepsilon,2})$	Polycrystal λ_s	($\times 10^{-6}$)
			3d Metals			
BCC-Fe	26	−30	21	−21	−7	
HCP-Co[u]	(−150)	(45)	(−140)	(50)	(−62)	
FCC-Ni	−60	−35	−46	−24	−34	
BCC-FeCo	—	—	140	30	—	
a-$Fe_{80}B_{20}$	48 (isotropic)	—	—	—	+32	
a-$Fe_{40}Ni_{40}B_{20}$	+20	—	—	—	+14	
a-$Cos_{80}B_{20}$	−4	—	—	—	−4	
			4f Metals/Alloys			
Gd[u]	(−175)	(105)	(−10)	0	—	
Tb[u]	—	(8700)	—	(30)	—	
$TbFe_2$	—	4400	—	2600	1753	
Terfenol $Tb_{0.3}Dy_{0.7}Fe_2$	—	—	—	1600	1200	**K_u is small**
			Spinel Ferrites			
Fe_3O_4	0	50	−15	56	+40	
$MnFe_2O_4$[u]	—	—	(−54)	(10)	—	
$CoFe_2O_4$	—	—	−670	120	−110	
			Garnets			
YIG	−0.6	−2.5	−1.4	−1.6	−2	
			Hard Magnets			
$Fe_{14}Nd_2B$[u]	—	—	—	—	—	
$BaO\cdot 6Fe_3O_4$[u]	—	—	(13)	—	—	

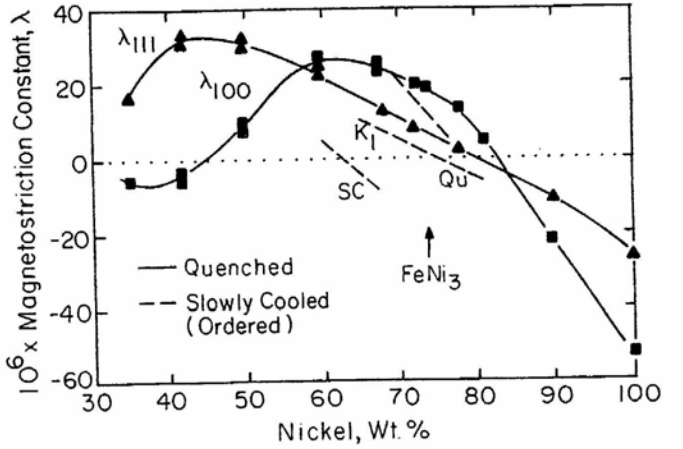
Composition dependence of magnetostrictions

Polycrystalline FCC FeNi and amorphous alloys RT



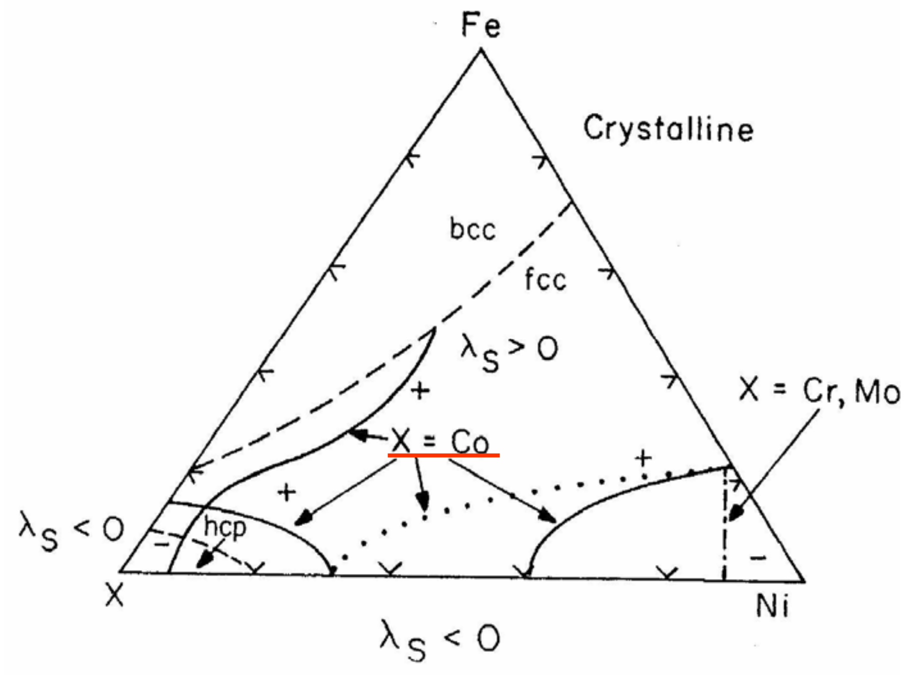
Due to BCC-FCC transformation, Invar alloys

FCC FeNi RT



Magnetostriction for Fe-Ni-X (X = Co, Cr, Mo)

Bold and dot/dashed lines exhibit zero magnetostriction observed by different researchers.

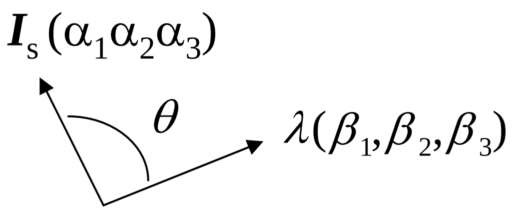


Zero magnetostriction materials are important for soft magnetic materials applied for magnetic heads etc.

Phenomenology

Cubic structure

$$\lambda = \Delta l / l = \frac{3}{2}\lambda_{100}(\alpha_1^2\beta_1^2 + \alpha_2^2\beta_2^2 + \alpha_3^2\beta_3^2 - \frac{1}{3})$$
$$+3\lambda_{111}(\alpha_1\alpha_2\beta_1\beta_2 + \alpha_2\alpha_3\beta_2\beta_3 + \alpha_3\alpha_1\beta_3\beta_1) \qquad (2.2.1)$$
$$\alpha_1^2 + \alpha_2^2 + \alpha_3^2 = 1$$
$$\beta_1^2 + \beta_2^2 + \beta_3^2 = 1$$



For $\alpha_i = \beta_i$

$$\lambda = \lambda_{100} + 3(\lambda_{111} - \lambda_{100})(\alpha_1^2\alpha_2^2 + \alpha_2^2\alpha_3^2 + \alpha_3^2\alpha_1^2) \qquad (2.2.2)$$

***E*x.1) H//[100]**

By inserting $\alpha_1 = 1$, $\alpha_2 = \alpha_3 = 0$ to (2.21), we obtain $\lambda = \frac{3}{2}\lambda_{100}(\beta_1^2 - \frac{1}{3})$.

Thus, the strain parallel to [100] is obtained for $\beta_1 = 1$; $\lambda = \lambda_{100}$

While, the strain parallel to [010] perpendicular to [100] is given by $\beta_1 = 0$

$$\lambda_{010} = \frac{3}{2}\lambda_{100}(0 - \frac{1}{3}) = -\frac{1}{2}\lambda_{100} \qquad \text{; shrink}$$

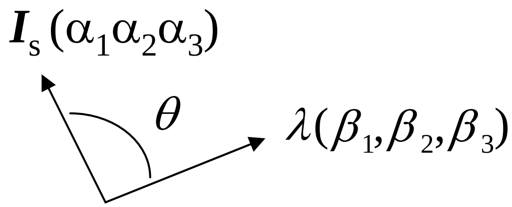
***E*x.2) H//[110]**

By inserting $\alpha_1 = \alpha_2 = 1/\sqrt{2}, \beta_1 = \beta_2 = 1/\sqrt{2}, \alpha_3 = \beta_3 = 0$ to (2.21),

$$\lambda_{110} = \frac{1}{4}(\lambda_{100} + 3\lambda_{111}) \qquad (2.2.3)$$

***Ex.3*) Polycrystalline**

It is isotropic. $\lambda_{100} = \lambda_{111} = \lambda_s$



From (2.2.1) we obtain

$$\lambda = \frac{3}{2}\lambda_s[(\alpha_1\beta_1 + \alpha_2\beta_2 + \alpha_3\beta_3)^2 - \frac{1}{3}]$$

While,

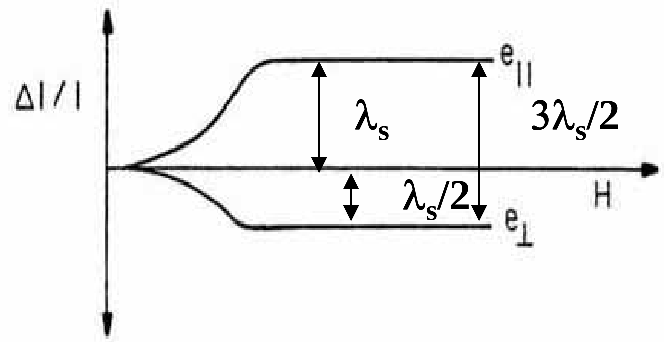
$$\cos\theta = \alpha_1\beta_1 + \alpha_2\beta_2 + \alpha_3\beta_3$$

Thus,

$$\lambda = \frac{3}{2}\lambda_s(\cos^2\theta - \frac{1}{3}) \qquad (2.2.4)$$

This is uniaxial.

$$\left.\begin{array}{l} \lambda = \lambda_s \text{ for } \theta = 0 \\ \lambda = -\lambda_s/2 \text{ for } \theta = \pi/2 \end{array}\right\} \longrightarrow \lambda_s = \frac{2}{3}(e_{//} - e_{\perp})$$

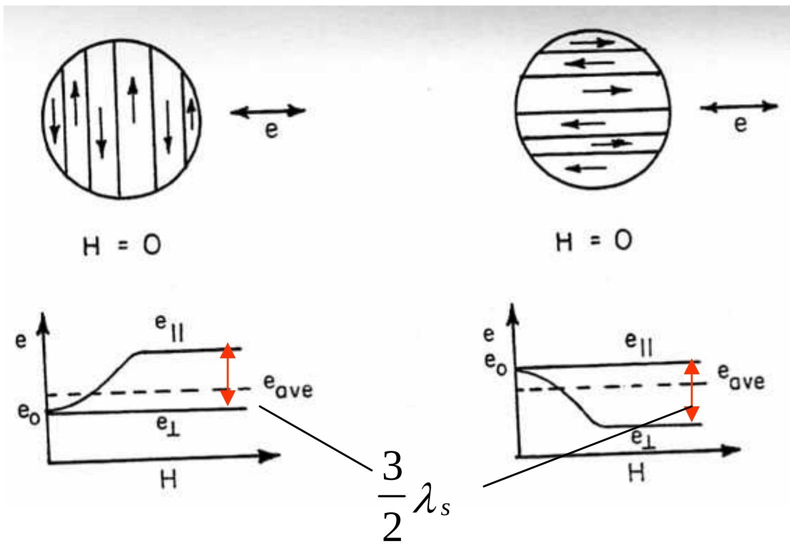


The saturation magnetostriction λ_s can be measured in the direction of the applied magnetic field, stating with a sample in the randomly magnetized state. However, because a completely demagnetized state is not easily achieved, **measurement of both $e_{//}$ and $e_{\perp}$ is recommended to determine λ_s in isotropic materials.**

Magnetostriction for a polycrystalline sample with 180 ° domain wall only.

The figure shows two limiting initial domain states that differ in the orientation of their easy-axis directions relative to the strain sensing direction. The sample at left is in an initial state of contraction while that at the right is in extension, when measured as indicated.

There is no magnetoresistive shape change associated with a magnetization process involving Only 180 ° domain walls ($e_{\perp}$at left or $e_{//}$ and at right). Consequently, a strain of $3\lambda_s/2$ is measured in magnetizing a sample with initial transverse magnetization ($e_{//}$ at left or $e_{\perp}$at right). This illustrates the importance of the initial state when strain in being measured or used in a device.



_Ex._4) Completely random polycrystalline materials

From (2.2.1)

$$\lambda = \Delta l / l = \frac{3}{2}\lambda_{100}(\alpha_1^2\beta_1^2 + \alpha_2^2\beta_2^2 + \alpha_3^2\beta_3^2 - \frac{1}{3})$$
$$+3\lambda_{111}(\alpha_1\alpha_2\beta_1\beta_2 + \alpha_2\alpha_3\beta_2\beta_3 + \alpha_3\alpha_1\beta_3\beta_1)$$

For $\alpha_i = \beta_i$ in (2.2.1) and considering that

$$\alpha_1^2\alpha_2^2 + \alpha_2^2\alpha_3^2 + \alpha_3^2\alpha_1^2 = \sin^4\theta\cos^2\varphi\sin^2\varphi + \sin^2\theta\cos^2\theta$$

averaging (2.2.1) over 3 dimensional space gives

$$\lambda_s = \int_0^{\pi}\sin\theta d\theta\int_0^{2\pi} d\varphi[Eq.(2.2.1)] = \frac{2}{5}\lambda_{100} + \frac{3}{5}\lambda_{111} \qquad (2.2.5)$$

2.2.2 Magnetoelastic energy

When there is strain, the stress σ is induced. Thus, the self elastic energy is given by

$$E_\sigma = \frac{1}{2}\lambda\sigma$$

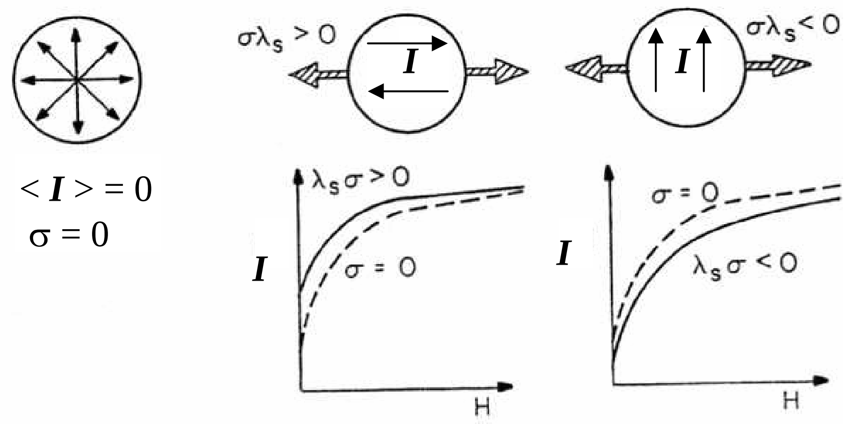
For polycrystalline we obtain from (2.24)

$$E_\sigma = \frac{3}{4}\lambda_s\sigma(\cos^2\theta - \frac{1}{3})$$

When external stress is applied, the elastic energy is given by $E_\sigma = -\lambda\sigma$

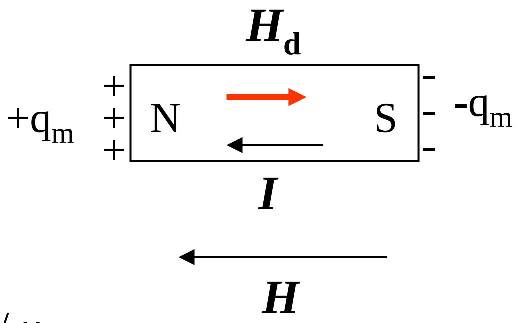
For polycrystalline, $E_\sigma = -\frac{3}{2}\lambda_s\sigma(\cos^2\theta - \frac{1}{3})$

Stressing or straining a magnetic material can produce a change in its preferred magnetization direction as shown in the figure. If λ_s is positive, it is easier to magnetize the material in the tensile stress (σ > 0) direction. It is harder to magnetize a material in a direction for which $\lambda_s < 0$ and $\sigma > 0$ or for which $\lambda_s > 0$ and $\sigma < 0$, namely $\lambda_s\sigma < 0$.



2.3 Magnetic domain and domain structures

Shape anisotropy



$\mathrm{div}\boldsymbol{B} = 0$

Demagnetization field: $\boldsymbol{H}_d$

$\boldsymbol{H}_d = - N_d\, \boldsymbol{I} / \mu_0$

N_d: Demagnetization coefficient, dependent on the shape of the magnetic material

Magnetostatic energy:

$$E_d = -\frac{1}{2}\mathbf{I}\mathbf{H}_d = \frac{1}{2\mu_0} N_d I^2$$

Reduction of magnetostatic energy

In order to reduce the magnetostatic energy multi-domain structure is formed.

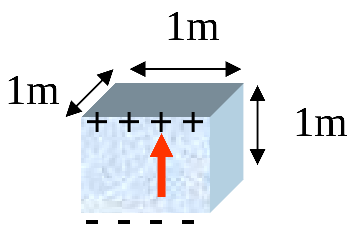
For example

Fe

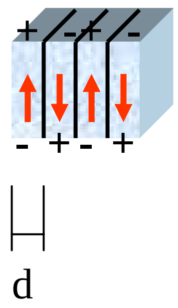
$I_s = 2.2$ Wb/m^2

$N_d = 1$ is assumed

(a) Single domain



(b) Multi-domain



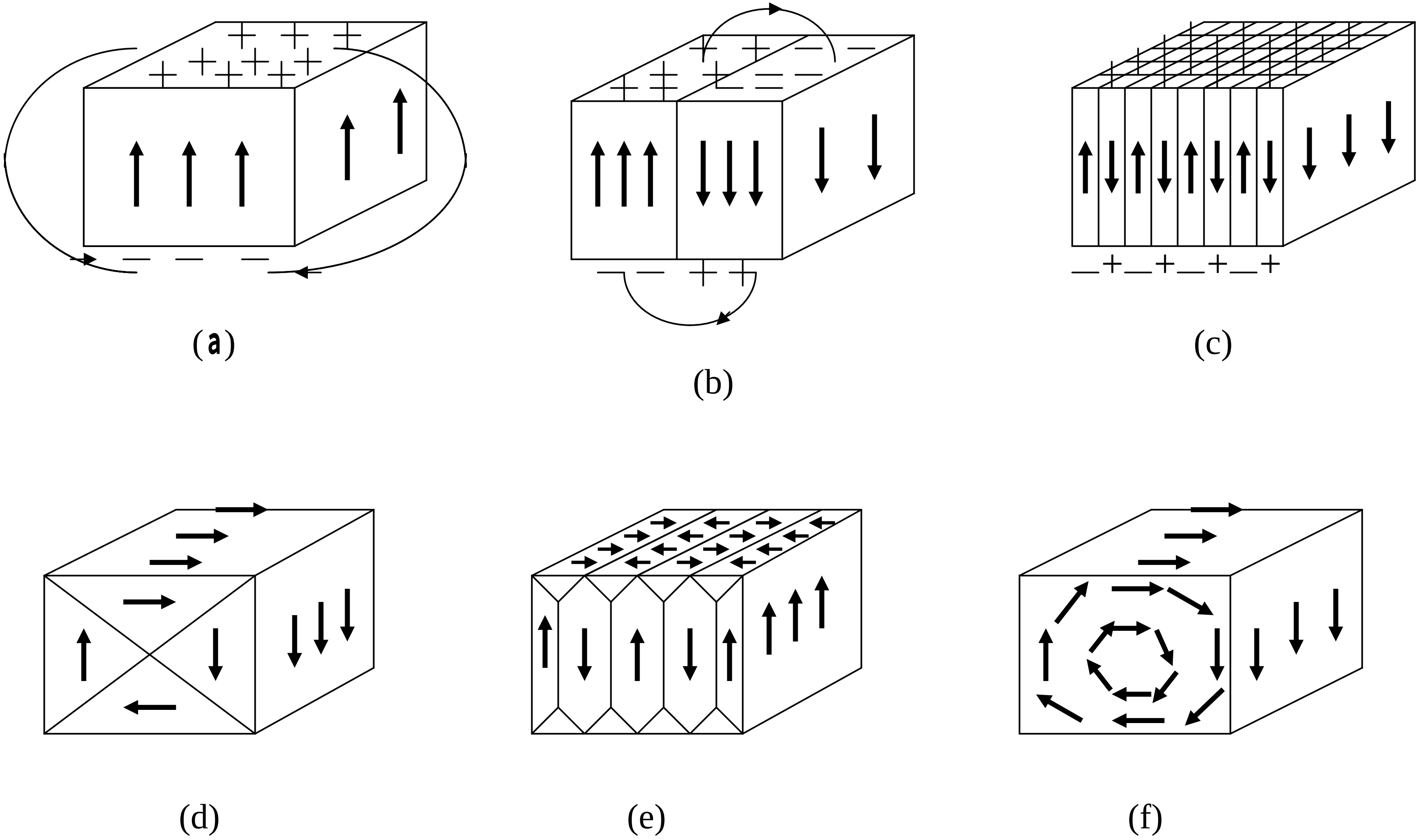
Magnetostatic energy per unit volume

(a) $E_d = N_d I^2 / 2\mu_0 = 1 \times 2.2^2 / 2 \times 4\pi \times 10^{-7} = 1.9 \times 10^6 [J/m^3]$

(b) $E_d = \frac{1}{d} N_d (Id)^2 / 2\mu_0 = 1.9 \times 10^6 d [J/m^3]$

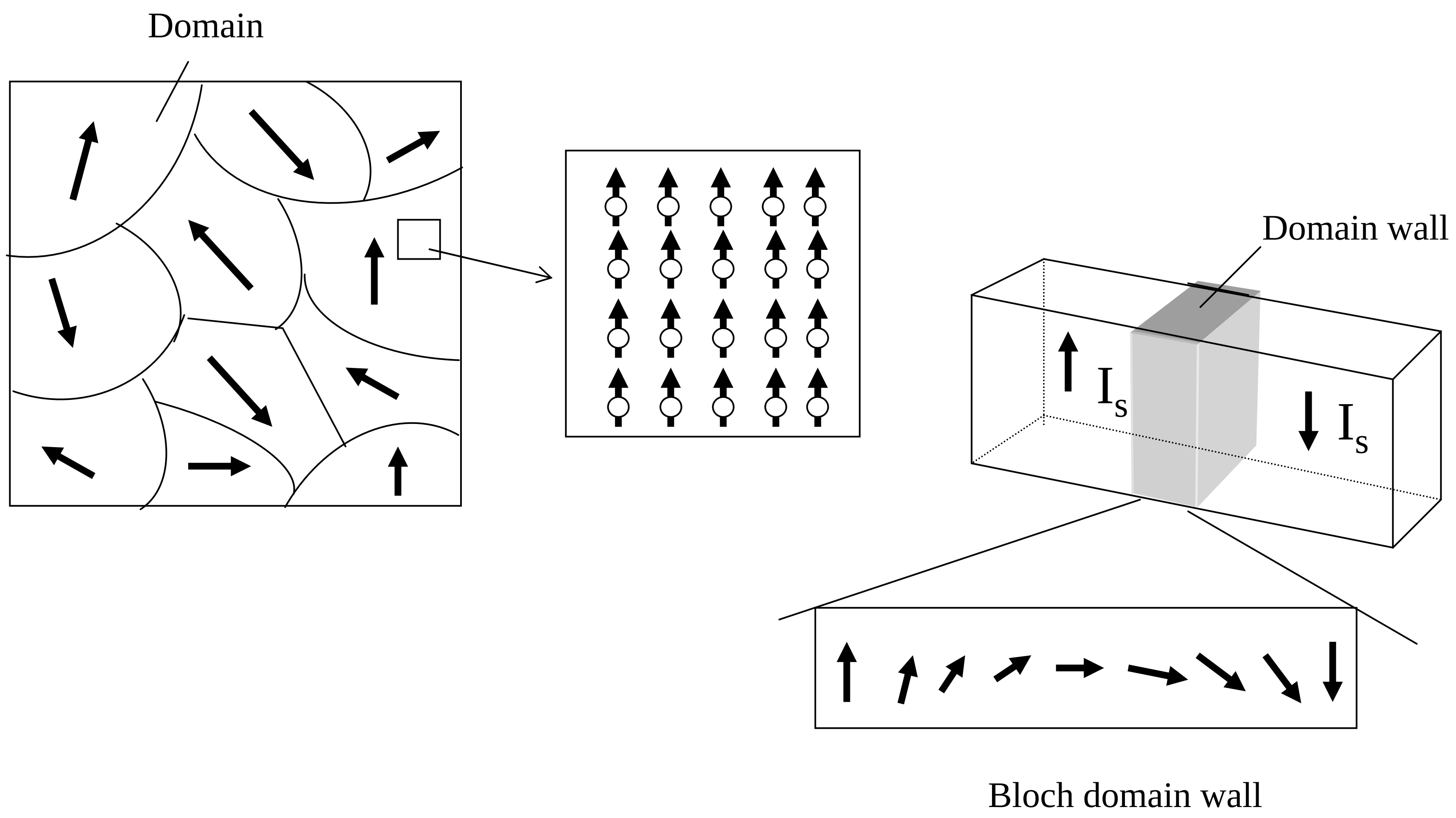
E_d(b) < E_d (a)

Domain structures



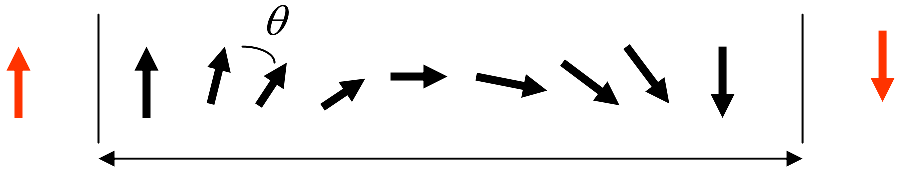
Exchange and anisotropy energies increase

Magnetic domain wall



Domain wall thickness and energy

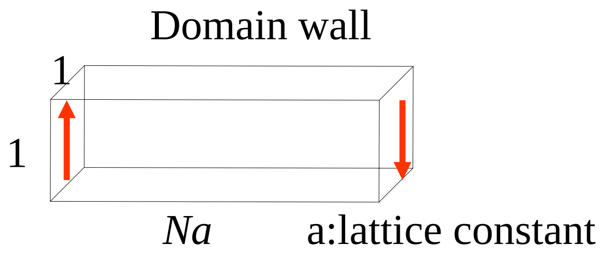
Consider 180° domain wall.



Domain wall width, $\delta = Na$

a is the lattice constant, N is the number of atoms in the domain wall.

The domain wall thickness is determined by minizing the exchange energy and magnetic anisotropy energy.



(1) Exchange energy: $E_{ex} = -2J\sum_{i,j}\mathbf{S}_i\mathbf{S}_j$

Exchange energy **per unit area** of domain wall : U_{ex}

$$U_{ex} = E_{ex} \times M = -2JS^2\cos\theta(N/a^2)$$

where, $\underline{M = (1/a) \times (1/a) \times N}$ is the number of atoms per unit area of domain wall.

If we assume that θ is small, $\cos\theta \approx 1-\theta^2$

$$U_{ex} = JS^2\theta^2(N/a^2) = JS^2\pi^2/Na^2 \quad \text{for} \quad \theta \approx \pi/N \qquad (2.3.1)$$

(2) Magnetic anisotropy energy (uniaxial)

$$E_A = K_u \sum_i \sin^2\theta_i$$

Anisotropy energy **per unit area** of domain wall : U_A

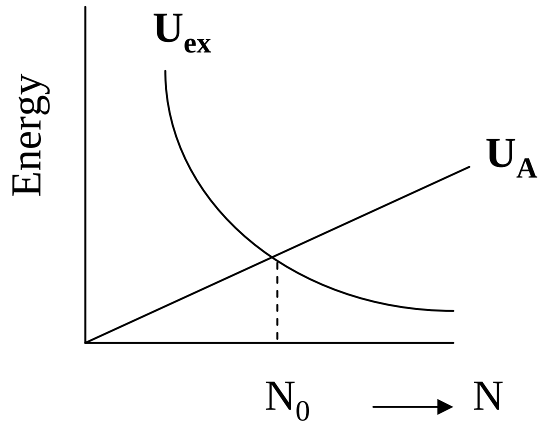
$$U_A = E_A a = K_u a(\sin^2\varphi + \sin^2 2\varphi + \sin^2 3\varphi + \cdots + \sin^2\pi) \qquad \varphi = \pi / N$$

$$= K_u a \sum_n \sin^2 n\varphi = K_u a \sum_n \sin^2(\frac{\pi}{N} n)$$

When N is large, Σ can be replaced by the integration.

$$U_A = K_u a \int_0^N \sin^2(\frac{\pi}{N}) n dn = K_u a \frac{N}{\pi} \int_0^\pi \sin^2(\frac{\pi}{N}) n d(\frac{\pi}{N} n)$$

$$= K_u a \frac{N}{\pi} \frac{\pi}{2} = \frac{1}{2} K_u a N \qquad (\int_0^\pi \sin^2 x dx = \frac{\pi}{2} \text{ is used}) \qquad (2.3.2)$$



From (2.3.1) and (2.3.2) the domain wall energy per unit area is given by

$$\gamma = U_{ex} + U_{ex} = J S^2 \pi^2 / N a^2 + \frac{1}{2} K_u a N$$

$\partial\gamma / \partial N = 0$ gives

$$N = \sqrt{2JS^2\pi^2 / K_u a^3}$$

Domain wall thickness is given by $\delta = Na = \sqrt{2}\pi\sqrt{JS^2 / aK_u}$

$$= \sqrt{2}\pi\sqrt{A / K_u} \qquad (2.3.3)$$

where, $A = JS^2 / a$

Domain wall energy per unit area is given by $\gamma = \sqrt{2}\pi\sqrt{AK_u}$ (2.3.4)

For Co
$A = 1.3 \times 10^{-11} [J / m]$
$K_{u1} = 4.53 \times 10^5 [J / m^3]$
$\delta = 1.68 \times 10^{-8} m = 0.168 nm$

Micromagnetics

The assumption of the uniform rotation of spins in a domain wall is not correct. In order to determine the domain wall parameters, θ **must be allowed to be a function of position**. In a one-dimensional approach to the problem, the local volume energy density at any position z along the normal to the Bloch wall is given by a sum of anisotropy and exchange terms:

$$f = \frac{F}{V} = f_a(\theta) + A\left(\frac{\partial\theta}{\partial z}\right)^2 \qquad (2.3.5)$$

The total surface energy density of the domain wall is

$$\gamma = \int_{-\infty}^{\infty} \left[f_a(\theta) + A\left(\frac{\partial\theta}{\partial z}\right)^2 \right] dz \qquad (2.3.6)$$

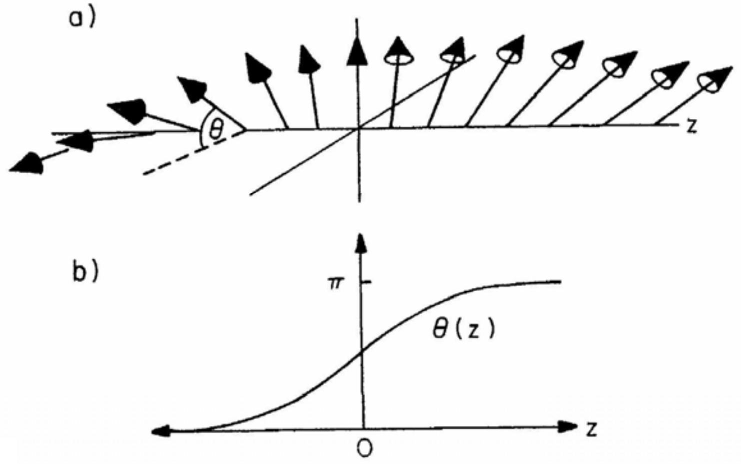


Figure 8.4 (*a*) Magnified sketch of the spin orientations within a 180° Bloch wall i uniaxial material; (*b*) an approximation of the variation of *θ* with distance *z* thro the wall.

In order to calculate the stable wall profile function $\theta(z)$, γ is minimized with respect to variation of the wall profile $\delta\theta(z)$

$$\delta\gamma = \int_{-\infty}^{\infty} [\frac{\partial f_a(\theta)}{\partial\theta}\,\delta\theta + 2A\frac{\partial\theta}{\partial z}\ \frac{\partial\delta\theta}{\partial z}]dz = 0$$

Integrating the second term by pars gives

$$\delta\gamma = \int_{-\infty}^{\infty} [\frac{\partial f_a(\theta)}{\partial\theta} - 2A\frac{\partial^2\theta}{\partial z^2}]\delta\theta dz + \left|2A\frac{\partial\theta}{\partial z}\delta\theta\right|_{-\infty}^{\infty} = 0$$

The last term is zero because $\frac{\partial\theta}{\partial z}$. vanishes far from the wall where the magnetic moments are fixed in orientation by the anisotropy axis inside the domain.

$\delta\theta(z)$ is an arbitrary function of z,

$$\frac{\partial f_a(\theta)}{\partial\theta} - 2A\frac{\partial^2\theta}{\partial z^2} = 0 \qquad (2.3.7)$$

The first and second terms in (2.3.7) are the local torque on a spin due to the gradient in anisotropy and in exchange energy at each point, respectively.

For a uniaxial material $f_a(\theta) = K_u \sin^2\theta$ and thus eqs. (2.3.7) gives

$$z = \sqrt{A/K_u}\ \ln[\tan(\theta/2)]$$

$$\longrightarrow \quad \theta(z) = 2\arctan[\exp(\pi z/\delta)]$$

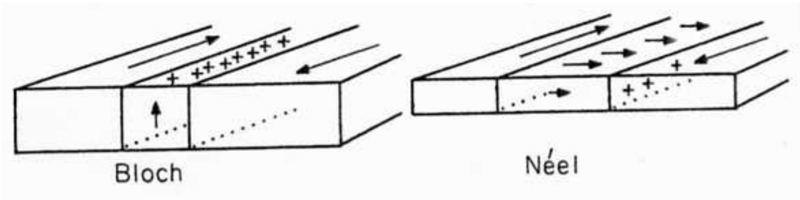
where $\delta = \pi\sqrt{A/K_u}$: wall thickness (2.3.8)

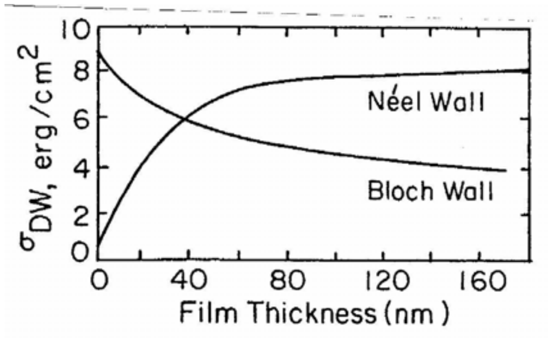
Domain wall energy per unit area is given by

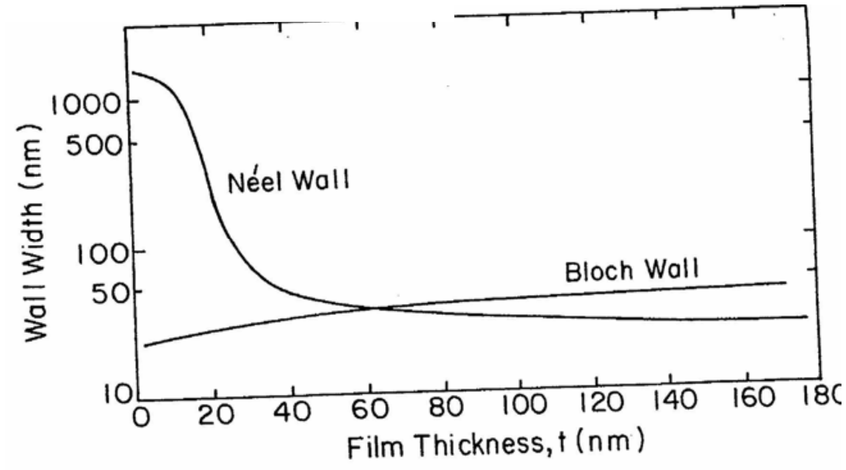
$$\gamma = 4\sqrt{AK_u} \qquad (2.3.9)$$

Neel wall

As sample thickness decreases, the magnetostatic energy of the wall increases as a result of the free poles at the top and bottom of the wall. To reduce this magnetostatic energy, the spins inside the wall may execute their 180 ° rotation in such a way as to minimize their magnetostatic energy, which leads to the rotation of spins in the plane of the surface. Such a wall is called a Neel wall.







A = 10^{-11} J/m, B_s = 1 T and K = 100 J/m^3

Domains in fine particles

It is important to know the size below which a particle is composed of a single domain for permanent magnets and recording media.

To a first approximation, it may be assumed that the critical particle diameter would be comparable to the domain wall width, that is, in particle of diameter **d <** $\delta = \pi\sqrt{A/K_u}$ there can be no domain wall present. Such a approach, however, does not take account of the magnetostatic energy that drives domain formation.

For the single domain state to be stable, the energy needed to create a domain wall spanning a spherical particle of radius r, namely, $\gamma\pi r^2 = 4\pi r^2\sqrt{AK_u}$ must be exceed the magnetostatic energy $\frac{1}{3}\mu_0 I_s^2 V = \frac{4}{9}\mu_0 I_s^2 \pi r^3$. Thus **the critical radius of the sphere for the single domain** is

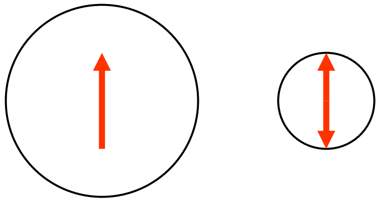
$$r_c \approx 9\sqrt{AK_u} / \mu_0 I_s^2 \qquad (2.3.10)$$

For Fe $r_c \approx 3nm$

For $SmCo_5$ with $K_u = 1 \times 10^7$ J/m^3 $r_c \approx 2-3\mu m$

Superparamagnetism

The magnetic particle size cannot be reduced indefinitely with useful magnetic properties. **Below a certain size the remanent magnetization is no longer fixed in the direction of the anisotropy**. Ambient thermal energy may be large enough to cause the moment to jump between two different stable orientation of magnetizations.



$VK_u > k_B T$ $\quad$ $VK_u < k_B T$

The probability of the magnetization revarsal is

$$I(t) = I_s e^{-t/\tau} \qquad \tau = \tau_0 e^{VK_u/k_B T} \qquad \tau_0 \approx 10^{-9}\ \text{sec}$$

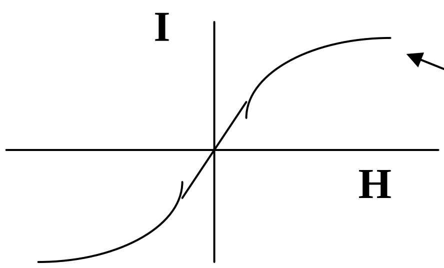
For $\tau \approx 10^2\ \text{sec}$ $\qquad V_c K_u \approx 25 k_B T \longrightarrow$ **The larger K_u the smaller V_c**

The time-averaged magnetization appears to be zero for $V < V_c$, while the magnetization is essentially uniform over the particle volume at any time. Such a magnetism is called as **superparamagnetism**.

The I-H curve of superparamagnetism is resemble that of ferromagnets but with two distinguishing features: (1) the approach to saturation follows a Langevin behavior and (2) there is no coercivity.

For Co particle with $K_u = 4.1 \times 10^5\ \text{J/m}^3$ the critical diameter for superparamagnetism is

$r_c \approx 4nm$



Langevin function

L (*x*) = coth *x* - 1/*x*